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Mark Scheme (Results)

Summer 2016

Pearson Edexcel GCE in Core Mathematics 3 (6665/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given
   - $\vdash$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q),$ where $|pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q),$ where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks
1(a)	$\text{fg}(x) = \dfrac{28}{x-2} - 1 \qquad \left(= \dfrac{30-x}{x-2}\right)$	M1
	Sets $\text{fg}(x) = x \Rightarrow \dfrac{28}{x-2} - 1 = x$	
	$\Rightarrow 28 = (x+1)(x-2)$	
	$\Rightarrow x^2 - x - 30 = 0$	M1
	$\Rightarrow (x-6)(x+5) = 0$	
	$\Rightarrow x = 6\,,\, x = -5$	dM1 A1 **(4)**
(b)	$a = 6$	B1 ft **(1)** **5 marks**
Alt 1(a)	$\text{fg}(x) = x \Rightarrow \text{g}(x) = \text{f}^{-1}(x)$	M1
	$\dfrac{4}{x-2} = \dfrac{x+1}{7}$	
	$\Rightarrow x^2 - x - 30 = 0$	M1
	$\Rightarrow (x-6)(x+5) = 0$	
	$\Rightarrow x = 6\,,\, x = -5$	dM1 A1 **4 marks**
S. Case	Uses $\text{gf}(x)$ instead $\text{fg}(x)$	Makes an error on $\text{fg}(x)$
	$\dfrac{4}{7x-1-2} = x$	Sets $\text{fg}(x) = x \Rightarrow \dfrac{7\times 4}{7\times(x-2)} - 1 = x$ M0
	$\Rightarrow 7x^2 - 3x - 4 = 0$	$\Rightarrow x^2 - x - 6 = 0$ M1
	$\Rightarrow (7x+4)(x-1) = 0$	$\Rightarrow (x+2)(x-3) = 0$
	$\Rightarrow x = -\dfrac{4}{7}, \quad x = 1$	$\Rightarrow x = -2, \quad x = 3$ dM1 A0 **2 out of 4 marks**

(a)

M1 Sets or implies that $\text{fg}(x) = \dfrac{28}{x-2} - 1$ Eg accept $\text{fg}(x) = 7\left(\dfrac{4}{x-2}\right) - 1$ followed by $\text{fg}(x) = \dfrac{7\times 4}{x-2} - 1$

Alternatively sets $\text{g}(x) = \text{f}^{-1}(x)$ where $\text{f}^{-1}(x) = \dfrac{x \pm 1}{7}$

Note that $\text{fg}(x) = 7\left(\dfrac{4}{x-2}\right) - 1 = \dfrac{28}{7(x-2)} - 1$ is M0

M1 Sets up a 3TQ (= 0) from an attempt at $\text{fg}(x) = x$ or $\text{g}(x) = \text{f}^{-1}(x)$

dM1 Method of solving 3TQ (= 0) to find at least one value for *x*. See "General Priciples for Core Mathematics" on page 3 for the award of the mark for solving quadratic equations
This is dependent upon the previous M. You may just see the answers following the 3TQ.

A1 Both $x = 6$ and $x = -5$

(b)

B1ft For $a = 6$ but you may follow through on the largest solution from part (a) provided more than one answer was found in (a). Accept 6, $a = 6$ and even $x = 6$
Do not award marks for part (a) for work in part (b).

Question	Scheme	Marks
2(a)	$y=\dfrac{4x}{\left(x^2+5\right)}\Rightarrow\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=\dfrac{4\left(x^2+5\right)-4x\times 2x}{\left(x^2+5\right)^2}$	M1A1
	$\Rightarrow\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=\dfrac{20-4x^2}{\left(x^2+5\right)^2}$	M1A1
		(4)
(b)	$\dfrac{20-4x^2}{\left(x^2+5\right)^2}<0\Rightarrow x^2>\dfrac{20}{4}$ Critical values of $\pm\sqrt{5}$	M1
	$x<-\sqrt{5}$, $x>\sqrt{5}$ or equivalent	dM1A1
		(3)
		7 marks

(a)M1 Attempt to use the **quotient rule** $\dfrac{vu'-uv'}{v^2}$ with $u=4x$ and $v=x^2+5$. If the rule is quoted it must be correct. It may be implied by their $u=4x,u'=A,\ v=x^2+5,v'=Bx$ followed by their $\dfrac{vu'-uv'}{v^2}$

If the rule is neither quoted nor implied only accept expressions of the form $\dfrac{A\left(x^2+5\right)-4x\times Bx}{\left(x^2+5\right)^2}, A,B>0$ You may condone missing (invisible) brackets

Alternatively uses the **product rule** with $u\left(/v\right)=4x$ and $v\left(/u\right)=\left(x^2+5\right)^{-1}$. If the rule is quoted it must be correct. It may be implied by their $u=4x,u'=A,\ v=x^2+5,v'=Bx\left(x^2+5\right)^{-2}$ followed by their $vu'+uv'$. If the rule is neither quoted nor implied only accept expressions of the form $A\left(x^2+5\right)^{-1}\pm 4x\times Bx\left(x^2+5\right)^{-2}$

A1 f$'(x)$ correct (unsimplified). For the product rule look for versions of $4\left(x^2+5\right)^{-1}-4x\times 2x\left(x^2+5\right)^{-2}$

M1 Simplifies to the form f$'(x)=\dfrac{A+Bx^2}{\left(x^2+5\right)^2}$ oe. This is not dependent so could be scored from $\dfrac{v'u-u'v}{v^2}$

When the product rule has been used the A of $A\left(x^2+5\right)^{-1}$ must be adapted.

A1 CAO. Accept exact equivalents such as $\left(\text{f}'(x)\right)=\dfrac{4\left(5-x^2\right)}{\left(x^2+5\right)^2}$, $-\dfrac{4x^2-20}{\left(x^2+5\right)^2}$ or $\dfrac{-4\left(x^2-5\right)}{x^4+10x^2+25}$

Remember to isw after a correct answer

(b)

M1 Sets their numerator either $=0$, <0, >0, and proceeds to at least **one** value for x

For example $20-4x^2..0\Rightarrow x..\sqrt{5}$ will be M1 dM0 A0.

It cannot be scored from a numerator such as 4 or indeed $20+4x^2$

dM1 Achieves **two** critical values for their numerator $=0$ and chooses the outside region

Look for $x<$ smaller root, $x>$ bigger root. Allow decimals for the roots.

Condone $x,,\ -\sqrt{5}, x..\sqrt{5}$ and expressions like $-\sqrt{5}>x>\sqrt{5}$

If they have $4x^2-20<0$ following an incorrect derivative they should be choosing the inside region

A1 Allow $x<-\sqrt{5}, x>\sqrt{5}$ $x<-\sqrt{5}$ or $x>\sqrt{5}$ $\{x:-\infty<x<-\sqrt{5}\cup\sqrt{5}<x<\infty\}$ $|x|>\sqrt{5}$

Do not allow for the A1 $x<-\sqrt{5}$ and $x>\sqrt{5}$. $\sqrt{5}<x<-\sqrt{5}$ or $\{x:-\infty<x<-\sqrt{5}\cap\sqrt{5}<x<\infty\}$ but you may isw following a correct answer.

Question	Scheme	Marks
3.(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=\frac{1}{2}\Rightarrow\alpha=26.57°$	M1A1
		(3)
(b)	$\frac{2}{2\cos\theta-\sin\theta-1}=15\Rightarrow\frac{2}{\sqrt{5}\cos(\theta+26.6°)-1}=15$	
	$\Rightarrow\cos(\theta+26.6°)=\frac{17}{15\sqrt{5}}=(awrt\ 0.507)$	M1A1
	$\theta+26.57°=59.54°$	
	$\Rightarrow\theta=awrt\ 33.0°$ or $awrt\ 273.9°$	A1
	$\theta+26.6°=360°-$ their $'59.5°'$	dM1
	$\Rightarrow\theta=awrt\ 273.9°$ and $awrt\ 33.0°$	A1
		(5)
(c)	$\theta-$ their $26.57°=$ their $59.54°\Rightarrow\theta=...$	M1
	$\theta=$ awrt $86.1°$	A1
		(2)
		(10 marks)

(a)

B1 $R=\sqrt{5}$. Condone $R=\pm\sqrt{5}$ Ignore decimals

M1 $\tan\alpha=\pm\frac{1}{2}$, $\tan\alpha=\pm\frac{2}{1}\Rightarrow\alpha=...$

If their value of R is used to find the value of α only accept $\cos\alpha=\pm\frac{2}{R}$ OR $\sin\alpha=\pm\frac{1}{R}\Rightarrow\alpha=...$

A1 $\alpha=$ awrt $26.57°$

(b)

M1 Attempts to use part (a) $\Rightarrow\cos(\theta\pm\text{their }26.6°)=K$, $|K|\,,,\,1$

A1 $\cos(\theta\pm\text{their }26.6°)=\frac{17}{15\sqrt{5}}=(\text{awrt }0.507)$. Can be implied by $(\theta\pm\text{their }26.6°)=\text{awrt }59.5°/59.6°$

A1 One solution correct, $\theta=awrt\ 33.0°$ or $\theta=awrt\ 273.9°$ Do not accept 33 for 33.0.

dM1 Obtains a second solution in the range. It is dependent upon having scored the previous M.
Usually for $\theta\pm\text{their }26.6°=360°-\text{their }59.5°\Rightarrow\theta=...$

A1 Both solutions $\theta=awrt\ 33.0°$ and a$wrt\ 273.9°$. Do not accept 33 for 33.0.
Extra solutions inside the range withhold this A1. Ignore solutions outside the range $0\,,,\,\theta<360°$

(c)

M1 $\theta-$ their $26.57°=$ their $59.54°\Rightarrow\theta=...$
Alternatively $-\theta+$ their $26.6°=-$their $59.5°\Rightarrow\theta=...$
If the candidate has an incorrect sign for α, for example they used $\cos(\theta-26.57°)$ in part (b) it would be scored for $\theta+$ their $26.57°=$ their $59.54°\Rightarrow\theta=...$

A1 awrt $86.1°$ ONLY. Allow both marks following a correct (a) and (b)
They can restart the question to achieve this result. Do not award if 86.1 was their smallest answer in (b). This occurs when they have $\cos(\theta-26.57°)$ instead of $\cos(\theta+26.57°)$ in (b)

Answers in radians: Withhold only one A mark, the first time a solution in radians appears
FYI (a) $\alpha=0.46$ (b) $\theta_1=$ awrt 0.58 and $\theta_2=$ awrt 4.78 (c) $\theta_3=$ awrt 1.50. Require 2 dp accuracy

Question	Scheme	Marks
4.(a)	(i) 21	B1
	(ii) $4e^{2x}-25=0 \Rightarrow e^{2x}=\frac{25}{4} \Rightarrow 2x=\ln\left(\frac{25}{4}\right) \Rightarrow x=\frac{1}{2}\ln\left(\frac{25}{4}\right), \Rightarrow x=\ln\left(\frac{5}{2}\right)$	M1A1, A1
	(iii) 25	B1
		(5)
(b)	$4e^{2x}-25=2x+43 \Rightarrow e^{2x}=\frac{1}{2}x+17$	M1
	$\Rightarrow 2x=\ln\left(\frac{1}{2}x+17\right) \Rightarrow x=\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$	A1*
		(2)
(c)	$x_1=\frac{1}{2}\ln\left(\frac{1}{2}\times 1.4+17\right)=awrt\ 1.44$	M1
	awrt $x_1=1.4368, x_2=1.4373$	A1
		(2)
(d)	Defines a suitable interval 1.4365 and 1.4375	M1
	...and substitutes into a suitable function Eg $4e^{2x}-2x-68$, obtains correct values with both a reason and conclusion	A1
		(2)
		(11 marks)

In part (a) accept points marked on the graph. If they appear on the graph and in the text, the text takes precedence. If they don't mark (a) as (i) (ii) and (iii) mark in the order given. If you feel unsure then please use the review system and your team leader will advise.

(a) (i)

B1 Sight of 21. Accept $(0,21)$

Do not accept just $|4-25|$ or $(21,0)$

(a) (ii)

M1 Sets $4e^{2x}-25=0$ and proceeds via $e^{2x}=\frac{25}{4}$ or $e^{x}=\frac{5}{2}$ to $x=..$

Alternatively sets $4e^{2x}-25=0$ and proceeds via $\left(2e^{x}-5\right)\left(2e^{x}+5\right)=0$ to $e^{x}=..$

A1 $\frac{1}{2}\ln\left(\frac{25}{4}\right)$ or awrt 0.92

A1 cao $\ln\left(\frac{5}{2}\right)$ or $\ln 5-\ln 2$. Accept $\left(\ln\left(\frac{5}{2}\right),0\right)$

(a) (iii)

B1 $k=25$ Accept also 25 or $y=25$

Do not accept just $|-25|$ or $x=25$ or $y=\pm 25$

(b)

M1 Sets $4e^{2x}-25=2x+43$ and makes e^{2x} the subject. Look for $e^{2x}=\frac{1}{4}(2x+43+25)$ condoning sign slips.Condone $\left|4e^{2x}-25\right|=2x+43$ and makes $\left|e^{2x}\right|$ the subject. Condone for both marks a solution with $x=a/\alpha$

An acceptable alternative is to proceed to $2e^{2x}=x+34\Rightarrow \ln 2+2x=\ln(x+34)$ using ln laws

A1* Proceeds correctly without errors to the correct solution. This is a given answer and the bracketing must be correct throughout. The solution must have come from $4e^{2x}-25=2x+43$ with the modulus having been taken correctly.

Allow $e^{2x}=\frac{1}{4}(2x+43+25)$ going to $x=\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$ without explanation

Allow $\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$ appearing as $\frac{1}{2}\log_e\left(\frac{1}{2}x+17\right)$ but not as $\frac{1}{2}\log\left(\frac{1}{2}x+17\right)$

.....

If a candidate attempts the solution backwards they must proceed from

$x=\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)\Rightarrow e^{2x}=\frac{1}{2}x+17\Rightarrow 4e^{2x}-25=2x+43$ for the M1

For the A1 it must be tied up with a minimal statement that this is $g(x)=2x+43$

.....

(c)

M1 Subs 1.4 into the iterative formula in an attempt to find x_1

Score for $x_1=\frac{1}{2}\ln\left(\frac{1}{2}\times 1.4+17\right)$ $x_1=\frac{1}{2}\ln(17.7)$ or awrt 1.44

A1 awrt $x_1=1.4368$, $x_2=1.4373$ Subscripts are not important, mark in the order given please.

(d)

M1 For a suitable interval. Accept 1.4365 and 1.4375 (or any two values of a smaller range spanning the root=1.4373) Continued iteration is M0

A1 Substitutes both values into **a suitable function**, which must be defined or implied by their working calculates both values correctly to 1 sig fig (rounded or truncated)

Suitable functions could be $\pm\left(4e^{2x}-2x-68\right)$, $\pm\left(x-\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)\right)$, $\pm\left(2x-\ln\left(\frac{1}{2}x+17\right)\right)$.

Using $4e^{2x}-2x-68$ f (1.4365) = -0.1, f (1.4375) = +0.02 or +0.03

Using $2e^{2x}-x-34$ f (1.4365) = -0.05/-0.06, f (1.4375) = +0.01

Using $x-\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$ f (1.4365) =-0.0007 or -0.0008, f (1.4375) =+ 0.0001 or +0.0002

Using $2x-\ln\left(\frac{1}{2}x+17\right)$ f (1.4365) = -0.001 or -0.002, f (1.4375) =+0.0003 or +0.0004

and states a reason (eg change of sign)

and a gives a minimal conclusion (eg root or tick)

It is valid to compare the two functions. Eg $\begin{matrix} g(1.4365)=45.7(6)<2\times 1.4365+43=45.8(73) \\ g(1.4375)=45.90>2\times 1.4375+43=45.8(75) \end{matrix}$

but the conclusion should be $g(x)=2x+43$ in between, hence root .

Similarly candidates can compare the functions x and $\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$

Question	Scheme	Marks
5 (i)	$y = \mathrm{e}^{3x}\cos 4x \Rightarrow \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) = \cos 4x \times 3\mathrm{e}^{3x} + \mathrm{e}^{3x} \times -4\sin 4x$	M1A1
	Sets $\cos 4x \times 3\mathrm{e}^{3x} + \mathrm{e}^{3x} \times -4\sin 4x = 0 \Rightarrow 3\cos 4x - 4\sin 4x = 0$	M1
	$\Rightarrow x = \dfrac{1}{4}\arctan\dfrac{3}{4}$	M1
	$\Rightarrow x =$ awrt 0.9463 4dp	A1
		(5)
(ii)	$x = \sin^2 2y \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = 2\sin 2y \times 2\cos 2y$	M1A1
	Uses $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\dfrac{\mathrm{d}x}{\mathrm{d}y} = 2\sin 4y \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2\sin 4y} = \dfrac{1}{2}\mathrm{cosec}\,4y$	M1A1
		(5) **(10 marks)**
(ii) Alt I	$x = \sin^2 2y \Rightarrow x = \dfrac{1}{2} - \dfrac{1}{2}\cos 4y$	2nd M1
	$\dfrac{\mathrm{d}x}{\mathrm{d}y} = 2\sin 4y$	1st M1 A1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2\sin 4y} = \dfrac{1}{2}\mathrm{cosec}\,4y$	M1A1
		(5)
(ii) Alt II	$x^{\frac{1}{2}} = \sin 2y \Rightarrow \dfrac{1}{2}x^{-\frac{1}{2}} = 2\cos 2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1A1
	Uses $x^{\frac{1}{2}} = \sin 2y$ AND $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2\sin 4y} = \dfrac{1}{2}\mathrm{cosec}\,4y$	M1A1
		(5)
(ii) Alt III	$x^{\frac{1}{2}} = \sin 2y \Rightarrow 2y = \mathrm{invsin}\, x^{\frac{1}{2}} \Rightarrow 2\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\sqrt{1-x}} \times \dfrac{1}{2}x^{-\frac{1}{2}}$	M1A1
	Uses $x^{\frac{1}{2}} = \sin 2y$, $\sqrt{1-x} = \cos 2y$ and $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2\sin 4y} = \dfrac{1}{2}\mathrm{cosec}\,4y$	M1A1
		(5)

(i)

M1 Uses the product rule $uv' + vu'$ to achieve $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) = A\mathrm{e}^{3x}\cos 4x \pm B\mathrm{e}^{3x}\sin 4x \quad A, B \neq 0$

The product rule if stated must be correct

A1 Correct (unsimplified) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \cos 4x \times 3\mathrm{e}^{3x} + \mathrm{e}^{3x} \times -4\sin 4x$

M1 Sets/implies their $\frac{dy}{dx}=0$ factorises/cancels)by e^{3x} to form a trig equation in just $\sin 4x$ and $\cos 4x$

M1 Uses the identity $\frac{\sin 4x}{\cos 4x}\equiv\tan 4x$, moves from $\tan 4x=C, C\neq 0$ using correct order of operations to $x=...$ Accept $x=$ awrt 0.16 (radians) $x=$ awrt 9.22 (degrees) for this mark.

If a candidate elects to pursue a more difficult method using $R\cos(\theta+\alpha)$, for example, the minimum expectation will be that they get (1) the identity correct, and (2) the values of R and α correct to 2dp. So for the correct equation you would only accept $5\cos(4x+\text{awrt } 0.93)$ or $5\sin(4x-\text{awrt } 0.64)$ before using the correct order of operations to $x=...$

Similarly candidates who square $3\cos 4x-4\sin 4x=0$ then use a Pythagorean identity should proceed from either $\sin 4x=\frac{3}{5}$ or $\cos 4x=\frac{4}{5}$ before using the correct order of operations …

A1 $\Rightarrow x=$ awrt 0.9463 .

Ignore any answers outside the domain. Withhold mark for additional answers inside the domain

(ii)

M1 Uses chain rule (or product rule) to achieve $\pm P\sin 2y\cos 2y$ as a derivative.

There is no need for lhs to be seen/ correct

If the product rule is used look for $\cancel{\frac{dx}{dy}}=\pm A\sin 2y\cos 2y\pm B\sin 2y\cos 2y$,

A1 Both lhs and rhs correct (unsimplified) . $\frac{dx}{dy}=2\sin 2y\times 2\cos 2y=(4\sin 2y\cos 2y)$ or

$1=2\sin 2y\times 2\cos 2y\frac{dy}{dx}$

M1 Uses $\sin 4y=2\sin 2y\cos 2y$ in their expression.

You may just see a statement such as $4\sin 2y\cos 2y=2\sin 4y$ which is fine.

Candidates who write $\cancel{\frac{dx}{dy}}=A\sin 2x\cos 2x$ can score this for $\cancel{\frac{dx}{dy}}=\frac{A}{2}\sin 4x$

M1 Uses $\frac{dy}{dx}=1\Big/\frac{dx}{dy}$ **for their expression in *y***. Concentrate on the trig identity rather than the coefficient in awarding this. Eg $\frac{dx}{dy}=2\sin 4y\Rightarrow\frac{dy}{dx}=2\text{cosec}\,4y$ is condoned for the M1

If $\frac{dx}{dy}=a+b$ do not allow $\frac{dy}{dx}=\frac{1}{a}+\frac{1}{b}$

A1 $\frac{dy}{dx}=\frac{1}{2}\text{cosec}\,4y$ If a candidate then proceeds to write down incorrect values of p and q then do not withhold the mark.

NB: See the three alternatives which may be less common but mark in exactly the same way. If you are uncertain as how to mark these please consult your team leader.

In **Alt I** the second M is for writing $x=\sin^2 2y\Rightarrow x=\pm\frac{1}{2}\pm\frac{1}{2}\cos 4y$ from $\cos 4y=\pm 1\pm 2\sin^2 2y$

In **Alt II** the first M is for writing $x^{\frac{1}{2}}=\sin 2y$ and differentiating both sides to $Px^{-\frac{1}{2}}=Q\cos 2y\cancel{\frac{dy}{dx}}$ oe

In **Alt 111** the first M is for writing $2y=\text{invsin}\left(x^{0.5}\right)$oe and differentiating to $M\cancel{\frac{dy}{dx}}=N\frac{1}{\sqrt{1-\left(x^{0.5}\right)^2}}\times x^{-0.5}$

Question	Scheme	Marks
6(a)	$x^2+x-6\overline{)x^4+x^3-3x^2+7x-6}$ $\quad x^2+3$ $x^4+x^3-6x^2$ $3x^2+7x-6$ $3x^2+3x-18$ $4x+12$	M1 A1
	$\dfrac{x^4+x^3-3x^2+7x-6}{x^2+x-6} \equiv x^2+3+\dfrac{4(x+3)}{(x+3)(x-2)}$	M1
	$\equiv x^2+3+\dfrac{4}{(x-2)}$	A1
		(4)
(b)	$f'(x)=2x-\dfrac{4}{(x-2)^2}$	M1A1ft
	Subs $x=3$ into $f'(x=3)=2\times3-\dfrac{4}{(3-2)^2}=(2)$	M1
	Uses $m=-\dfrac{1}{f'(3)}=\left(-\dfrac{1}{2}\right)$ with $(3, f(3))=(3,16)$ to form eqn of normal	
	$y-16=-\dfrac{1}{2}(x-3)$ or equivalent cso	M1A1
		(5)
		(9 marks)

(a)

M1 Divides $x^4+x^3-3x^2+7x-6$ by x^2+x-6 to get a quadratic quotient and a linear or constant remainder. To award this look for a minimum of the following

$$x^2+x-6\overline{)x^4+x^3-3x^2+7x-6} \quad \boldsymbol{x^2(+..x)+A}$$
$$\boldsymbol{x^4+x^3-6x^2}$$
$$\boldsymbol{(Cx)+D}$$

If they divide by $(x+3)$ first they must then divide their by result by $(x-2)$ before they score this method mark. Look for a cubic quotient with a constant remainder followed by a quadratic quotient and a constant remainder

Note: FYI Dividing by $(x+3)$ gives x^3-2x^2+3x-2 and $\left(x^3-2x^2+3x-2\right)\div\left(x-2\right)=x^2+3$ with a remainder of 4.

Division by $(x-2)$ first is possible but difficult.....please send to review any you feel deserves credit.

A1 Quotient = x^2+3 and Remainder = $4x+12$

M1 Factorises x^2+x-6 and writes their expression in the appropriate form.

$$\left(\frac{x^4+x^3-3x^2+7x-6}{x^2+x-6}\right) \equiv \text{Their Quadratic Quotient} + \frac{\text{Their Linear Remainder}}{(x+3)(x-2)}$$

It is possible to do this part by partial fractions. To score M1 under this method the terms must be correct and it must be a full method to find both "numerators"

A1 $x^2+3+\dfrac{4}{(x-2)}$ or $A=3, B=4$ but don't penalise after a correct statement.

(b)

M1 $x^2+A+\dfrac{B}{x-2}\to 2x\pm\dfrac{B}{(x-2)^2}$

If they fail in part (a) to get a function in the form $x^2+A+\dfrac{B}{x-2}$ allow candidates to pick up this method mark for differentiating a function of the form $x^2+Px+Q+\dfrac{Rx+S}{x\pm T}$ using the quotient rule oe.

A1ft $x^2+A+\dfrac{B}{x-2}\to 2x-\dfrac{B}{(x-2)^2}$ oe. FT on their numerical *A*, *B* for for $x^2+A+\dfrac{B}{x-2}$ only

M1 Subs $x=3$ into their $\mathrm{f}'(x)$ in an attempt to find a numerical gradient

M1 For the correct method of finding an equation of a normal. The gradient must be $-\dfrac{1}{\text{their f}'(3)}$ and the point must be $(3,\mathrm{f}(3))$. Don't be overly concerned about how they found their f(3), ie accept *x*=3 y =.

Look for $y-\mathrm{f}(3)=-\dfrac{1}{\mathrm{f}'(3)}(x-3)$ or $\big(y-\mathrm{f}(3)\big)\times-\mathrm{f}'(3)=(x-3)$

If the form $y=mx+c$ is used they must proceed as far as *c* =

A1 cso $y-16=-\dfrac{1}{2}(x-3)$ oe such as $2y+x-35=0$ but remember to isw after a correct answer.

Alt (a) attempted by equating terms.

Alt (a)	$x^4+x^3-3x^2+7x-6\equiv(x^2+A)(x^2+x-6)+B(x+3)$	M1
	Compare 2 terms (or substitute 2 values) AND solve simultaneously ie	M1
	$x^2\Rightarrow A-6=-3,\quad x\Rightarrow A+B=7,\quad$ const $\Rightarrow$ –6A+3B=– 6	
	$A=3,\ B=4$	A1,A1

1st Mark M1 Scored for multiplying by (x^2+x-6) and cancelling/dividing to achieve
$x^4+x^3-3x^2+7x-6\equiv(x^2+A)(x^2+x-6)+B(x\pm3)$

3rd Mark M1 Scored for comparing two terms (or for substituting two values) **AND** solving simultaneously to get values of ***A*** and ***B***.

2nd Mark A1 Either $A=3$ or $B=4$. One value may be correct by substitution of say $x=-3$

4th Mark A1 Both $A=3$ and $B=4$

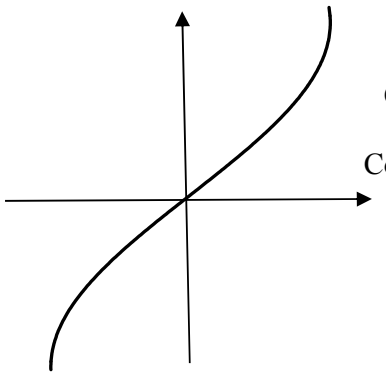
Alt (b) is attempted by the quotient (or product rule)

ALT (b)	$\mathrm{f}'(x)=\dfrac{\big(x^2+x-6\big)\big(4x^3+3x^2-6x+7\big)-\big(x^4+x^3-3x^2+7x-6\big)(2x+1)}{\big(x^2+x-6\big)^2}$	M1A1
1st 3 marks	Subs $x=3$ into	M1

M1 Attempt to use the **quotient rule** $\dfrac{vu'-uv'}{v^2}$ with $u=x^4+x^3-3x^2+7x-6$ and $v=x^2+x-6$ and achieves an expression of the form $\mathrm{f}'(x)=\dfrac{\big(x^2+x-6\big)\big(..x^3.....\big)-\big(x^4+x^3-3x^2+7x-6\big)(..x..)}{\big(x^2+x-6\big)^2}$.

Use a similar approach to the product rule with $u=x^4+x^3-3x^2+7x-6$ and $v=\big(x^2+x-6\big)^{-1}$

Note that this can score full marks from a partially solved part (a) where $\mathrm{f}(x)\equiv x^2+3+\dfrac{4x+12}{x^2+x-6}$

Question	Scheme	Marks
7(a)	 Correct position **or** curvature Correct position **and** curvature	M1 A1 **(2)**
(b)	$3\arcsin(x+1)+\pi=0 \Rightarrow \arcsin(x+1)=-\frac{\pi}{3}$ $\Rightarrow (x+1)=\sin\left(-\frac{\pi}{3}\right)$ $\Rightarrow x=-1-\frac{\sqrt{3}}{2}$	M1 dM1A1 **(3)** **(5 marks)**

(a) Ignore any scales that appear on the axes

M1 Accept for the method mark
Either one of the two sections with correct curvature passing through (0,0),
Or both sections condoning dubious curvature passing through (0,0) (but do not accept any negative gradients)
Or a curve with a different range or an "extended range"
See the next page for a useful guide for clarification of this mark.

A1 A curve only in quadrants one and three passing through the point (0,0) with a gradient that is always positive. The gradient should appear to be approx ∞ at each end. If you are unsure use review
If range and domain are given then ignore.

(b)

M1 Substitutes $g(x+1)=\arcsin(x+1)$ in $3g(x+1)+\pi=0$ and attempts to make $\arcsin(x+1)$ the subject

Accept $\arcsin(x+1)=\pm\frac{\pi}{3}$ or even $g(x+1)=\pm\frac{\pi}{3}$. Condone $\frac{\pi}{3}$ in decimal form awrt1.047

dM1 Proceeds by evaluating $\sin\left(\pm\frac{\pi}{3}\right)$ and making x the subject.

Accept for this mark $\Rightarrow x=\pm\frac{\sqrt{3}}{2}\pm1$. Accept decimal such as -1.866

Do not allow this mark if the candidate works in mixed modes (radians and degrees)
You may condone invisible brackets for both M's as long as the candidate is working correctly with the function

A1 $-1-\frac{\sqrt{3}}{2}$ oe with no other solutions. Remember to isw after a correct answer

Be careful with single fractions. $-\frac{2-\sqrt{3}}{2}$ and $\frac{-2+\sqrt{3}}{2}$ are incorrect but $-\frac{2+\sqrt{3}}{2}$ is correct

Note: It is possible for a candidate to change $\frac{\pi}{3}$ to $60°$ and work in degrees for all marks

Question	Scheme	Marks
8 (a)	$2\cot 2x + \tan x \equiv \dfrac{2}{\tan 2x} + \tan x$	B1
	$\equiv \dfrac{(1-\tan^2 x)}{\tan x} + \dfrac{\tan^2 x}{\tan x}$	M1
	$\equiv \dfrac{1}{\tan x}$	M1
	$\equiv \cot x$	A1*
		(4)
(b)	$6\cot 2x + 3\tan x = \text{cosec}^2 x - 2 \Rightarrow 3\cot x = \text{cosec}^2 x - 2$	
	$\Rightarrow 3\cot x = 1 + \cot^2 x - 2$	M1
	$\Rightarrow 0 = \cot^2 x - 3\cot x - 1$	A1
	$\Rightarrow \cot x = \dfrac{3 \pm \sqrt{13}}{2}$	M1
	$\Rightarrow \tan x = \dfrac{2}{3 \pm \sqrt{13}} \Rightarrow x = ..$	M1
	$\Rightarrow x = 0.294, -2.848, -1.277, 1.865$	A2,1,0
		(6)
		(10 marks)
8 (a)alt 1	$2\cot 2x + \tan x \equiv \dfrac{2\cos 2x}{\sin 2x} + \tan x$	B1
	$\equiv 2\dfrac{\cos^2 x - \sin^2 x}{2\sin x \cos x} + \dfrac{\sin x}{\cos x}$	M1
	$\equiv \dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x} + \dfrac{\sin^2 x}{\sin x \cos x} \equiv \dfrac{\cos^2 x}{\sin x \cos x}$	M1
	$\equiv \dfrac{\cos x}{\sin x}$	
	$\equiv \cot x$	A1*
8 (a)alt 2	$2\cot 2x + \tan x \equiv 2\dfrac{(1-\tan^2 x)}{2\tan x} + \tan x$	B1M1
	$\equiv \dfrac{2}{2\tan x} - \dfrac{2\tan^2 x}{2\tan x} + \tan x$ or $\dfrac{(1-\tan^2 x) + \tan^2 x}{\tan x}$	
	$\equiv \dfrac{2}{2\tan x} = \cot x$	M1A1*
Alt (b)	$6\cot 2x + 3\tan x = \text{cosec}^2 x - 2 \Rightarrow \dfrac{3\cos x}{\sin x} = \dfrac{1}{\sin^2 x} - 2$	
	$\left(\times \sin^2 x\right) \Rightarrow 3\sin x \cos x = 1 - 2\sin^2 x$	M1
	$\Rightarrow \dfrac{3}{2}\sin 2x = \cos 2x$	M1A1
	$\Rightarrow \tan 2x = \dfrac{2}{3} \Rightarrow x = ..$	M1
	$\Rightarrow x = 0.294, -2.848, -1.277, 1.865$	A2,1,0
		(6)

(a)

B1 States or uses the identity $2\cot 2x = \dfrac{2}{\tan 2x}$ or alternatively $2\cot 2x = \dfrac{2\cos 2x}{\sin 2x}$

This may be implied by $2\cot 2x = \dfrac{1-\tan^2 x}{\tan x}$. Note $2\cot 2x = \dfrac{1}{2\tan 2x}$ is B0

M1 Uses the correct double angle identity $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

Alternatively uses $\sin 2x = 2\sin x\cos x$, $\cos 2x = \cos^2 x - \sin^2 x$ oe and $\tan x = \dfrac{\sin x}{\cos x}$

M1 Writes their two terms with a single common denominator and simplifies to a form $\dfrac{ab}{cd}$.

For this to be scored the expression must be in either sin *x* and cos *x* or just tan *x*.

In alternative 2 it is for splitting the complex fraction into parts and simplifying to a form $\dfrac{ab}{cd}$.

You are awarding this for a correct method to proceed to terms like $\dfrac{\cos^2 x}{\sin x\cos x}$, $\dfrac{2\cos^3 x}{2\sin x\cos^2 x}$, $\dfrac{2}{2\tan x}$

A1* cso. For proceeding to the correct answer. This is a given answer and all aspects must be correct including the consistent use of variables. If the candidate approaches from both sides there must be a conclusion for this mark to be awarded. Occasionally you may see a candidate attempting to prove $\cot x - \tan x \equiv 2\cot 2x$. This is fine but again there needs to be a conclusion for the A1*
If you are unsure of how some items should be marked then please use review

(b)

M1 For using part (a) and writing $6\cot 2x + 3\tan x$ as $k\cot x$, $k \neq 0$ in their equation (or equivalent)
WITH an attempt at using $\text{cosec}^2 x = \pm 1 \pm \cot^2 x$ to produce a quadratic equation in just $\cot x$ / $\tan x$

A1 $\cot^2 x - 3\cot x - 1 = 0$ The = 0 may be implied by subsequent working
Alternatively accept $\tan^2 x + 3\tan x - 1 = 0$

M1 Solves a 3TQ=0 in $\cot x$ (or tan) using the formula or any suitable method for their quadratic to find at least one solution. Accept answers written down from a calculator. You may have to check these from an incorrect quadratic. FYI answers are $\cot x =$ awrt $3.30, \quad -0.30$

Be aware that $\cot x = \dfrac{3 \pm \sqrt{13}}{2} \Rightarrow \tan x = \dfrac{-3 \pm \sqrt{13}}{2}$

M1 For $\tan x = \dfrac{1}{\cot x}$ and using arctan producing at least one answer for x in degrees or radians.
You may have to check these with your calculator.

A1 Two of $x = 0.294, -2.848, -1.277, 1.865$ (awrt 3dp) in radians or degrees.
In degrees the answers you would accept are (awrt 2dp) $x = 16.8°, 106.8°, -73.2°, -163.2°$

A1 All four of $x = 0.294, -2.848, -1.277, 1.865$ (awrt 3 dp) with no extra solutions in the range $-\pi \leqslant x \leqslant \pi$

See main scheme for Alt to (b) using Double Angle formulae still entered M A M M A A in epen

1st M1 For using part (a) and writing $6\cot 2x + 3\tan x$ as $k\cot x$, $k \neq 0$ in their equation (or equivalent)
then using $\cot x = \dfrac{\cos x}{\sin x}$, $\text{cosec}^2 x = \dfrac{1}{\sin^2 x}$ and $\times \sin^2 x$ to form an equation sin and cos

1st A1 For $\dfrac{3}{2}\sin 2x = \cos 2x$ or equivalent. **Attached to the next M**

2nd M1 For using both correct double angle formula

3rd M1 For moving from $\tan 2x = C$ to *x*=..using the correct order of operations.

Question	Scheme	Marks
9(a)	Subs $D=15$ and $t=4$ $\quad x=15\mathrm{e}^{-0.2\times4}=6.740\ (mg)$	M1A1 **(2)**
(b)	$15\mathrm{e}^{-0.2\times7}+15\mathrm{e}^{-0.2\times2}=13.754(mg)$	M1A1* **(2)**
(c)	$15\mathrm{e}^{-0.2\times T}+15\mathrm{e}^{-0.2\times(T+5)}=7.5$ $15\mathrm{e}^{-0.2\times T}+15\mathrm{e}^{-0.2\times T}\mathrm{e}^{-1}=7.5$ $15\mathrm{e}^{-0.2\times T}(1+\mathrm{e}^{-1})=7.5\Rightarrow \mathrm{e}^{-0.2\times T}=\dfrac{7.5}{15(1+\mathrm{e}^{-1})}$ $\mathrm{T}=-5\ln\left(\dfrac{7.5}{15\left(1+\mathrm{e}^{-1}\right)}\right)=5\ln\left(2+\dfrac{2}{\mathrm{e}}\right)$	M1 dM1 A1, A1 **(4)** **(8 marks)**

(a)

M1 Attempts to substitute both $D=15$ and $t=4$ in $x=D\mathrm{e}^{-0.2t}$
It can be implied by sight of $15\mathrm{e}^{-0.8}$, $15\mathrm{e}^{-0.2\times4}$ or awrt 6.7
Condone slips on the power. Eg you may see -0.02

A1 CAO 6.740 (mg) Note that 6.74 (mg) is A0

(b)

M1 Attempt to find the sum of two expressions with $D=15$ in both terms with t values of 2 and 7
Evidence would be $15\mathrm{e}^{-0.2\times7}+15\mathrm{e}^{-0.2\times2}$ or similar expressions such as $\left(15\mathrm{e}^{-1}+15\right)\mathrm{e}^{-0.2\times2}$
Award for the sight of the two numbers awrt **3.70** and awrt **10.05**, followed by their total awrt **13.75**
Alternatively finds the amount after 5 hours, $15\mathrm{e}^{-1}=$ awrt **5.52** adds the second dose = **15** to get a total of awrt **20.52** then multiplies this by $\mathrm{e}^{-0.4}$ to get awrt **13.75**.
Sight of 5.52+15=20.52 $\rightarrow$ 13.75 is fine.

A1* cso so both the expression $15\mathrm{e}^{-0.2\times7}+15\mathrm{e}^{-0.2\times2}$ and $13.754(mg)$ are required
Alternatively both the expression $\left(15\mathrm{e}^{-0.2\times5}+15\right)\times\mathrm{e}^{-0.2\times2}$ and $13.754(mg)$ are required.
Sight of just the numbers is not enough for the A1*

(c)

M1 Attempts to write down a correct equation involving T or t. Accept with or without correct bracketing
Eg. accept $15\mathrm{e}^{-0.2\times T}+15\mathrm{e}^{-0.2\times(T\pm5)}=7.5$ or similar equations $\left(15\mathrm{e}^{-1}+15\right)\mathrm{e}^{-0.2\times T}=7.5$

dM1 Attempts to solve their equation, dependent upon the previous mark, by proceeding to $\mathrm{e}^{-0.2\times T}=\ldots$
An attempt should involve an attempt at the index law $x^{m+n}=x^m\times x^n$ and taking out a factor of $\mathrm{e}^{-0.2\times T}$ Also score for candidates who make $\mathrm{e}^{+0.2\times T}$ the subject using the same criteria

A1 Any correct form of the answer, for example, $-5\ln\left(\dfrac{7.5}{15\left(1+\mathrm{e}^{-1}\right)}\right)$

A1 CSO $\mathrm{T}=5\ln\left(2+\dfrac{2}{\mathrm{e}}\right)$ Condone t appearing for T throughout this question.

Alt (c) using lns

(c)	$15e^{-0.2\times T}+15e^{-0.2\times(T+5)}=7.5$ $15e^{-0.2\times T}+15e^{-0.2\times T}e^{-1}=7.5$ $e^{-0.2\times T}(1+e^{-1})=0.5\Rightarrow -0.2\times T+\ln(1+e^{-1})=\ln 0.5$ $\Rightarrow T=\dfrac{\ln 0.5-\ln(1+e^{-1})}{-0.2},\Rightarrow T=5\ln\left(2+\dfrac{2}{e}\right)$	M1 dM1 A1, A1 **(4)** **(8 marks)**

You may see numerical attempts at part (c).

Such an attempt can score a maximum of two marks.

This can be achieved either by

Method One

1st Mark (Method): $15e^{-0.2\times T}+\text{awrt } 5.52e^{-0.2\times T}=7.5\Rightarrow e^{-0.2\times T}=\text{awrt } 0.37$

2nd Mark (Accuracy): $T=-5\ln(\text{awrt } 0.37)$ or awrt 5.03 or $T=-5\ln\left(\dfrac{7.5}{\text{awrt } 20.52}\right)$

Method Two

1st Mark (Method): $13.754e^{-0.2\times T}=7.5\Rightarrow T=-5\ln\left(\dfrac{7.5}{13.754}\right)$ or equivalent such as 3.03

2nd Mark (Accuracy): 3.03+2=5.03 Allow $-5\ln\left(\dfrac{7.5}{13.754}\right)+2$

Method Three (by trial and improvement)

1st Mark (Method): $15e^{-0.2\times 5}+15e^{-0.2\times 10}=7.55$ or $15e^{-0.2\times 5.1}+15e^{-0.2\times 10.1}=7.40$ or any value between
2nd Mark (Accuracy): Answer $T=5.03$.

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